Math 1C Quiz 1 Version 1
Fri Oct 7, 2016

NAME YOU ASKED TO BE CALLED IN CLASS:

SCORE: _____ / 30 POINTS

1.	NO CALCULATORS OR NOTES ALLOWED
2.	UNLESS STATED OTHERWISE, YOU MUST SIMPLIFY ALL ANSWERS
3.	SHOW PROPER CALCULUS LEVEL WORK / PROOFS TO JUSTIFY YOUR ANSWERS

SCORE: 12 / 12 PTS

Determine if each of the following converges or diverges.
If it converges, determine what it converges to.
If it diverges, write "DIVERGES".
Justify each answer using proper mathematical reasoning, algebra and/or calculus.

[a] $\sum_{n=1}^{\infty} 3^{\frac{1-n}{n}}$

$$= \sum_{n=1}^{\infty} 3^{\frac{1}{n}} \cdot \frac{1}{3}$$

$$= \frac{1}{3}\sum_{n=1}^{\infty} 3^{\frac{1}{n}}$$

$$\lim_{n\to\infty} 3^{\frac{1}{n}} = 3^0 = 1 \neq 0 \Rightarrow \frac{1}{3}\lim_{n\to\infty} 3^{\frac{1}{n}} = \frac{1}{3} \neq 0$$

1 1

Series $\sum_{n=1}^{\infty} 3^{\frac{1-n}{n}}$ Diverges.

1

[b] $\left\{\frac{\sin n}{e^n}\right\}$

$$\lim_{n\to\infty} \frac{\sin n}{e^n}$$

Since $-1 \le \sin n \le 1$

then $-\frac{1}{e^n} \le \frac{\sin n}{e^n} \le \frac{1}{e^n}$ 1

We have $\lim_{n\to\infty}\left(-\frac{1}{e^n}\right) = \lim_{n\to\infty}\left(\frac{1}{e^n}\right) = 0$ 1

By Squeeze Theorem, $\lim_{n\to\infty}\frac{\sin n}{e^n} = 0$ 1

Converges to 0

[c] $\left\{\frac{n}{\sqrt{1+4n^2}}\right\}$

$$\lim_{x\to\infty}\frac{x}{\sqrt{1+4x^2}}$$

$$= \lim_{x\to\infty}\frac{1}{\sqrt{\frac{1}{x^2}+4}}$$ 2

$= \frac{1}{2}$. Converges to $\frac{1}{2}$.

1

[d] $\sum_{n=1}^{\infty}\frac{2+2^{2n}}{5^n}$

$$= \sum_{n=1}^{\infty}\left[2\cdot\left(\frac{1}{5}\right)^n + \left(\frac{4}{5}\right)^n\right]$$

both geometry series.

$$\Rightarrow 2\sum_{n=1}^{\infty}\left(\frac{1}{5}\right)^n + \sum_{n=1}^{\infty}\left(\frac{4}{5}\right)^n$$

$$= 2\times\frac{\frac{1}{5}}{1-\frac{1}{5}} + \frac{\frac{4}{5}}{1-\frac{4}{5}}$$

1 1

$$= \frac{1}{2} + 4$$

Series Converges to $\frac{9}{4}$

$$= \frac{9}{2}$$ 1

Consider the following statements. SCORE: 3 / 3 PTS

(i) If $\{a_n\}$ has limit 0, then $\sum_{n=1}^{\infty} a_n$ is convergent X

(ii) If $\sum_{n=1}^{\infty} a_n$ is divergent, then $\{a_n\}$ diverges X

(iii) If $\{a_n\}$ is bounded, then $\{a_n\}$ converges

Which of the statements above are true ? **Circle the correct answer below.**

3

[a]	none are true	[b]	only (i) is true	[c]	only (ii) is true	[d]	only (iii) is true
[e]	only (i) and (ii) are true	[f]	only (i) and (iii) are true	[g]	only (ii) and (iii) are true	[h]	all are true

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Find all values of x for which $\sum_{n=0}^{\infty} 2^{n+1}(3-x)^n$ is convergent. **You do NOT need to find the sum.** SCORE: ____ / 4 PTS

$$\sum_{n=0}^{\infty} 2^{n+1}\cdot(3-x)^n.$$

$$= 2\sum_{n=0}^{\infty} (6-2x)^n$$

Series Converges when $|6-2x| \leq 1$ ✳

$\Rightarrow \quad -1 \leq 6-2x \leq 1$

$x \in [\frac{5}{2}, \frac{7}{2}]$ ✳

Determine if each sequence below is increasing, decreasing or neither. SCORE: ____ / 5 PTS

Justify each answer using proper mathematical reasoning and/or algebra.

Your solutions must NOT use derivatives.

[a] $\left\{\frac{5n-11}{2n-5}\right\}$ $a_1=2,\ a_2=1$, $a_3=4,\ a_4=3.$ $\frac{1}{2}$

$f(x)=\frac{5x-11}{2x-5}$

$f(x+1)-f(x)=\frac{5(x+1)-11}{2(x+1)-5}-\frac{5x-11}{2x-5}$

$=\frac{5x-6}{2x-3}-\frac{5x-11}{2x-5}$

$= \frac{-3}{(2x-3)(2x-5)}$

1.5 2.5

If $\frac{3}{2}<x<\frac{5}{2}$, $f(x+1)-f(x)>0 \Rightarrow f(x+1)>f(x)$. ie: $x=2$

if $x\in(-\infty,\frac{3}{2})\cup(\frac{5}{2},+\infty)$, $f(x+1)-f(x)<0 \Rightarrow f(x+1)<f(x)$.

Therefore, Sequence $\{a_n\}$ is decreasing if $n=1$ and $n\geq 3$ $(a_1 \& a_2)$ and is increasing iff $n=2$. $(a_2 \& a_3)$

✳

[b] $\left\{\frac{3n-5}{4n-3}\right\}$ ✳

$f(x)=\frac{3x-5}{4x-3}$ $f(x+1)-f(x)=\frac{3(x+1)-5}{4(x+1)-3}-\frac{3x-5}{4x-3}$

$=\frac{11}{(4x+1)(4x-3)}>0$ 1 1

for $x\in(-\infty,-\frac{1}{4})\cup(\frac{3}{4},+\infty)$.

Therefore, since $n\in\mathbb{Z}^+$, $f(x+1)-f(x)>0$

$\Rightarrow f(x+1)>f(x)$.

Sequence $\left\{\frac{3n-5}{4n-3}\right\}$ increases. 1